UNIT II: THE ANALYTICAL FRAMEWORK OF THE FEDERAL PUBLIC DEBT BENCHMARK

As we saw in Unit I, the definition of the optimal public debt composition is one element of the strategic planning process. Optimal composition (benchmark) represents the profile desired for the long-term debt structure and acts as a guide for elaborating the government's short and medium-term financing strategies.

In the Brazilian case, the benchmark is expressed by a set of indicators of relevance to the debt, including composition of outstanding debt by type of index, maturity structure, particularly the percentage of debt to mature in the next 12 months, and the average maturity of the outstanding debt. Implementation of the benchmark can be achieved through definition of targets for the values to be attained by these indicators over a specific temporal horizon.

This Unit is an effort to describe the model utilized by the National Treasury in evaluating the trade-offs between costs and risks derived from alternative profiles for the Federal Public Debt (FPD) structure over the long-term, based on the objectives and guidelines defined for management of that debt[1].

- This unit is organized into four sections:
- Initially, we will present the main theoretical arguments in favor of the adoption of a benchmark, together with information on international experience in this area;
- In section 2, we will describe the simulation model used by the National Treasury;
- Application of the model for defining the FPD benchmark is illustrated in section 3;
- Finally, section 4 presents FPD composition desired for the long-term, in the form of upper and lower limits.

[18] Definition of a FPD benchmark was discussed in such previous studies as CABRAL & LOPES (2005), SILVA, CABRAL & BACHDASSARIAN (2006), CABRAL et alli. (2008) and ALVES (2009).

1. Literature and International Experience

The importance of the optimal composition (benchmark) is clearly supported by theoretical literature, which stresses the relevance of public debt management to economic activity[2]. This is particularly true regarding the literature on tax smoothing and temporal consistency, which supports active debt management. The theoretical arguments in favor of seeking an adequate debt composition take on even greater relevance when one considers the elements discussed in literature on the credibility of macroeconomic policies, signaling and real effects of a sovereign default, among others[3].

In this debate, one should also mention the contribution made by multilateral institutions, including the World Bank and International Monetary Fund. In their publication Guidelines for Public Debt Management (WB; IMF, 2001), these two institutions describe the benchmark as a powerful tool for representing the debt profile that the government desires to attain, based on its preferences as defined by the trade-off between costs and risks.

Finally, international experience has documented that various countries have taken measures to define an optimal composition for their debts. A case in point is Portugal, one of the pioneers in the formulation and adoption of optimal long-term composition for quantifying the objective of its public debt management, as well as for enhancing the consistency between daily decisions and the long-term objective. Denmark, Sweden, Canada and the United Kingdom have also developed models designed to aid in defining a benchmark portfolio to be used as a guide in elaboration of financing strategies. The following chart summarizes aspects of the experience of these countries.

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[2] The Hypothesis of Ricardian Equivalence is an important point of departure in this debate, though it does not provide subsidiary information for an active defense of public indebtedness. One consequence of the Hypothesis of Ricardian Equivalence is the neutrality of the debt with respect to economic activity, since debt and taxes are equivalent from the intertemporal point of view. However, Ricardian Equivalence is basically founded upon the following suppositions: 1) an infinite planning horizon; 2) complete markets; and 3) taxes that do not cause distortions (BARRO, 1974; 1979; 1989). With relaxation of these presuppositions, new theories have drawn conclusions regarding the importance of adequate debt management.

[3] See GOLDFAJN & DE PAULA (1999).

Tabela 1. International Experience

Country	Relevant indicator	Summary of the model	Source
Portugal	Cost and risk of cash flows, with restrictions regarding refinancing risk	The model assumes a constant nominal debt in a stationary state and has three inputs: 1) stochastic interest rates; s) financing strategies; and 3) deterministic scenarios for other macroeconomic variables. A subset of the best solutions of the model is presented to the authorities charged with the final decision.	IGCP (1999).
Sweden	Targets for the participation of each type of debt, based on cash flows and duration	Running Yield: probability distribution of the model is calculated through dynamic simulation of the interest (short and long-term), inflation, exchange rate and GDP curves. A measurement of dispersion of this distribution is used as risk indicator.	RIKSGÄLDEN (2008); RIKSGÄLDEN (2009)
Ireland	Current net value and fiscal volatility	The current net value is used as a cost measurement, and fiscal volatility as a risk measurement. Since it reflects the structural conditions of the economy and the final objective of fiscal policy, the benchmark should not be altered significantly over time. In this sense, revisions in the benchmark can be implemented to reflect structural changes in the economy, but not in response to short-term movements.	NTMA (2006); NTMA (2011)
Denmark	Duration of the portfolio	In order to define the duration target, a long-term analysis of the evolution of expected costs is carried out. The model considers only the domestic debt. Interest rate risk is dealt with in the ALM approach and the trade-off between costs and risks is evaluated by a Cost-at-Risk model. The model combines stochastic and deterministic scenarios.	DANMARKS NATIONALBANK (2007)
England	Debt Service Cost (in terms of cash flows) as a proportion of GDP	The benchmark is not utilized. However, models are utilized to illustrate the impact of different issuance strategies and indicators for risk management. Long-term analysis of financing strategies is done through the use of stochastic scenarios based on the combination of a macroeconomic model with specifications for interest curves. Cost is measured by cash flows and risk by the dispersion of payments.	UK-DMO (2011) e PICK AND ANTHONY (2006)
Canada	Cost measurement: annual average debt service burden as a percentage of total outstanding debt Measurement of risk: cost of volatility or consideration of impact on the budget	Combines a macroeconomic model with interest curves to simulate costs and risks of alternative financing strategies. In order to aggregate more than one objective into the analysis, the model offers a tool for minimizing the weight of the debt service with restrictions on other objectives.	BOLDER (2008)

Elaboration: National Treasury

2. The Brazilian Optimal Composition Model

2.1. Some Methodological Issues

The analytical framework utilized in the study of optimal FPD composition is based on stochastic simulations derived from finance and efficient portfolio theories. However, before proceeding to a description of the model itself, some comments must be made regarding the direct application of the instruments of traditional financial analysis to government policies.

In general, the government, with the condition of preserving prudent risk levels, may have more complex objectives than reducing costs. Aside from this, evolution of its cash flows and indicators of budget impacts may have implications for the choice of the optimal debt structure. One should also consider that, given the nature of the public debt, government measures have a strong impact on bond prices and, consequently, on the cost and risk of its financing strategies. As a result, these peculiarities may lead economic policy managers to define a debt composition as benchmark that is different from those on the efficient frontier, obtained from the strictly financial point of view.

One important question in the model refers to what should be the relevant debt concept for evaluating costs and risks. In the Brazilian case, the National Treasury only has direct control over the Federal Public Debt, which encompasses all domestically and internationally issued bonds, as well as the federal government's external contractual debt. Nonetheless, the most commonly used indicator, both by the federal government, to define its debt targets and the primary surplus required to achieve those targets, and by analysts, with the aim of evaluating fiscal sustainability, is the ratio between the Net Public Sector Debt and GDP (NPSD/GDP). This concept is more inclusive since it encompasses all public sector liabilities, deducted from its assets against other economic agents. Public sector is understood as the federal government (including the Social Security System), Central Bank, state and municipal governments and public sector companies.

The reduction in NPSD volatility (risk) is important to the extent in which the occurrence of shocks with potential to jeopardize its sustainability requires a fiscal policy response. In this sense, unforeseen NPSD fluctuations may result in tax surprises that would affect the available income of the population, generating inefficiencies from the social welfare point of view[4]. Despite the fact that the National Treasury's work instrument is the FPD, there is clear communication between this debt and the NPSD, which is much broader and used as an economic policy reference.

For the reasons set out above, a decision was made to use the NPSD/GDP indicator for choosing the optimal composition in Brazil. This choice was based on the idea that, in an analysis of government intertemporal budget restrictions aimed at evaluating the sustainability of the public debt, all public sector assets and liabilities should be considered. As a matter of fact, many economic analysts and

[4] Literature on optimal taxation suggests that if taxes cause dead weight losses, the government should smooth them over time, thus minimizing distortions consequent upon revenue inflows. In this case, the debt profile and its risks are relevant for public policy purposes, since fluctuations in debt costs would result in alterations in the tax load. See BOHN (1990).

financial market participants, including international organizations (e.g., the World Bank and IMF) and rating agencies (e.g., Standard & Poor's), consider NPSD/GDP as the relevant indicator for evaluating the sustainability of the Brazilian debt.

Another important aspect refers to the fact that the benchmark study is based on the premises of the steady-state. This has two meanings for the model. In the first place, it presupposes that the economy is already in a steady state or, in other words, that all variables are fluctuating around their long-term equilibrium values. The fact of the matter is that this supposition is appropriate to a discussion of a debt profile desired for the long-term, avoiding the possibility of the decision being contaminated by transitory fluctuations in economic scenarios. In practical terms, the reference to the steady state scenario includes the following characteristics: stability of the economic environment, low fiscal vulnerability, real interest rates already in a state of equilibrium, controlled inflation and sustainable economic growth.

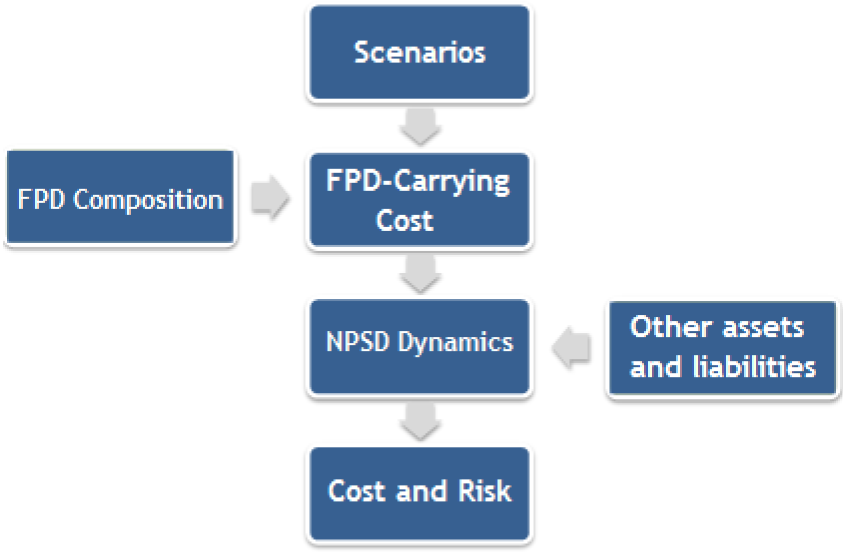
The second meaning found in the idea of a steady-state is that each issuance strategy implicitly preserves constant the characteristics in the long-term FPD portfolio. In other words, the strategy itself and, therefore, debt management guidelines must be stable over time, with no sharp fluctuations caused by temporary shocks in the economy, coupled with avoidance of nearsighted behavior guided by short-term parameters.

2.2. The Optimal Composition Model

The study of the optimal composition (benchmark) for the Brazilian debt is based upon the application of stochastic simulation methods, with the objective of deriving an efficient frontier of debt compositions, expressing potential trade-offs between costs and risks in FPD management. In this sense, a composition is viewed as efficient when it has the lowest risk for a specified cost level or, alternatively, when the cost is the lowest for a specified risk level. The entire set of compositions that satisfied this condition defines the efficient frontier, while it is the debt manager's task to choose which composition is desirable, since it is not possible to obtain simultaneous cost and risk reductions among the frontier portfolios.

The following Figure illustrates the general idea of the model used for analysis of the trade-off between costs and risks.

Figure 1. Schematic Summary of the Model



Source: National Treasury

Initially, various stochastic scenarios are generated for the principal macroeconomic and financial variables – output, inflation, exchange rates, short-term interest rates and public bond prices – with the objective of simulating evolution of the major factors that influence the trajectory and cost of the public debt over time.

At that point, a debt composition is chosen that involves a basket of securities, with varied types of indexes and different maturities. By way of example, we will look at a portfolio composed equally of one year fixed rate bonds and five-year floating rate bonds. Over the course of each simulated scenario, the FPD financing cost is calculated for the chosen debt composition.

The next step of the simulation process involves calculation of FPD and NPSD evolution which depends on the previously obtained FPD financing cost, as well as on other parameters that define the public sector primary result and evolution of "other assets and liabilities[5]" that make up the NPSD.

Finally, cost and risk indicators are derived from analysis of the behavior of NPSD in the face of stochastic shocks. Since thousands of scenarios are simulated and the value of the NPSD is calculated for each one of

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[5] FPD includes only the domestic and external debts for which the National Treasury is liable. In calculating NPSD, the definition of public sector used to measure indebtedness is that of the nonfinancial public sector plus the Central Bank. Consequently, it encompasses the direct federal, state and municipal administrations, indirect administrations, the public social security system, nonfinancial federal, state and municipal government companies, as well as the Central Bank of Brazil. In this concept, intragovernmental debts are excluded, so as to measure only the public sector debt with private agents. Since NPSD is a net debt concept, public sector liabilities are deducted from its assets with other economic agents.

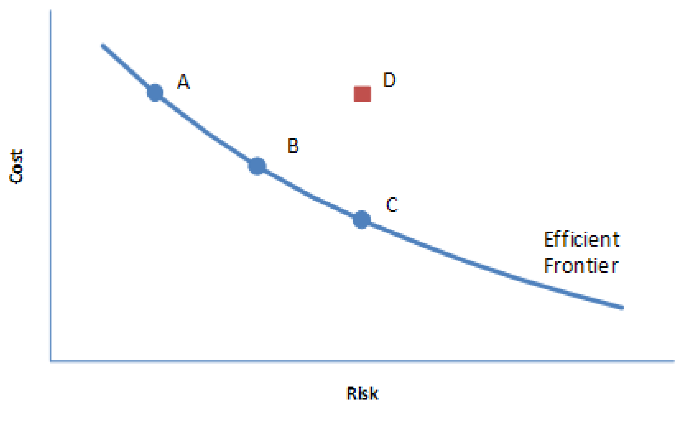
them for a given point in time, in practical terms the results provide a distribution of probabilities of the value of NPSD, from which the costs and risk metrics are extracted.

In the simulations, the initial FPD structure, stochastic scenarios, primary result parameters and "other assets and liabilities" are the same for any debt composition chosen. As a result, the only reason for the resulting trajectory of NPSD simulations to vary among different debt compositions is the selected composition itself.

After performing the simulations, the metrics obtained for each evaluated composition are plotted on a graph in which the axes are NPSD/GDP cost and risk, in such a way that the efficient frontier is obtained as a curve composed of the points that represent the lowest cost for a specified risk level. The frontier portfolios are efficient because it is not possible to alternate among these portfolios in order to obtain cost and risk reduction gains simultaneously. Finally, given the government's position on risk (which should reflect that of society), it is possible to choose a specific frontier portfolio that will define the debt benchmark.

The following figure illustrates this concept. Portfolios over the length of the frontier (A, B and C) are efficient because the risk necessarily increases when one seeks to reduce the cost of the debt alternating between these compositions (from A to B; or from B to C). Compositions above and to the right of the frontier are inefficient because they increase risk, given the level of costs (D compared to A), or increase the cost for a given risk level (D compared to C), or increase both the cost and the risk, in comparison to an efficient composition (D compared to B).

Figure 2. Illustration of the Efficient Frontier



Source: National Treasury

In operational terms, the efficient frontier depends on the average cost and standard deviation of each bond, just as occurs with the cost correlation matrix of these securities. In the first place, simulations based on debt compositions with 100% of a specified bond provide the average cost and standard deviation. Secondly, simulations based on portfolios with 50%-50% pairs of bonds aid in calculating the cost correlation matrix[6,7]. Consequently, with these data in hand, a numerical procedure is employed in order to discover the lowest cost portfolio for each possible risk level, concluding with the obtaining of the analytical efficient frontier.

In the case of the FPD benchmark, portfolios can be composed of four basic instruments which differ with respect to their characteristics: fixed rate bonds, floating rate bonds, inflation-linked bonds and bonds denominated in foreign currencies. Each one of these categories also differs in terms of maturities, thus making it possible to specify a basket of representative short, medium and long-term bonds. More specifically, the financing instruments considered are as follows:

- Fixed rate : 1, 3, 5 and 10 years;
- Floating-rate (indexed to the Selic rate): 5 years;
- Inflation-linked : 10 and 30 years;
- Denominated in foreign currency (exchange rate): 10 and 30 years.

Although the bond listing above seeks to reflect the financing options currently available for FPD financing, the simulation framework has the flexibility needed for inclusion of other securities, currencies and maturities. Besides this, in generating the efficient frontier it is possible to include such technical restrictions as a minimum (or maximum) percentage for participation of a security or category in the debt portfolio, or a minimum average maturity for the optimal portfolio.

Next, two large sets of procedures used to simulate the model are emphasized. In the first place, the generation of scenarios for simulating the dynamics of the economy and calculating the cost of debt financing are shown. Right after that, the dynamics of the debt from which the cost and risk indicators result in the model will be presented.

2.3. Dynamics of the Economy

The set of simulations depends on the generation of scenarios for the economic variables that determine the cost of debt financing and the dynamics of the NPSD/GDP ratio. With this, the model requires specification of a set of equations used to describe how these variables evolve over time.

[6] The correlation between the cost of two bonds is extracted through the following ratio: cost variance (σ_p^2) of a portfolio with two securities is $\sigma_p^2 = w_1^2\sigma_1^2 + (1-w_1)^2\sigma_2^2 + 2\rho_{12}\sigma_1\sigma_2$, in which σ_1 is the cost variance of security 1; σ_2 is the cost variance of security 2; ρ_{12} is the correlation between the costs of securities 1 and 2; $0 \leq w_1 \leq 1$ is the relative weight of security 1 in the portfolio.

[7] The correlation matrix must be semi-defined positive. When this does not occur, a spectral decomposition is utilized to obtain a semi-defined positive matrix that is close to the original. This decomposition considers that when a matrix is not semi-defined positive, it has at least one negative eigenvalue. The procedure for decomposition utilizes the positive eigenvalues of the original matrix and substitutes the negative values for zero in order to recompose the matrix. This method provides a reasonable approximation for the correlation matrix. The steps for spectral decomposition can be seen in JÄCKEL (2002).

In this sense, the basic processes of the model cover the following variables:

- Basic interest rate (Selic);
- Term structure of interest rates for:;
  - fixed-rate securities;
  - IPCA-linked securities;
  - securities denominated in foreign currency;
- Inflation rates (domestic and foreign);
- Exchange rate (real and nominal[8]);
- Gross Domestic Product (GDP).

Aside from the deterministic component, the equations that describe the evolution of the variables above add a stochastic term with the objective of simulating random shocks in their trajectories. One assumes that these stochastic shocks follow a correlation structure, thus conferring macroeconomic consistency on the simulations.

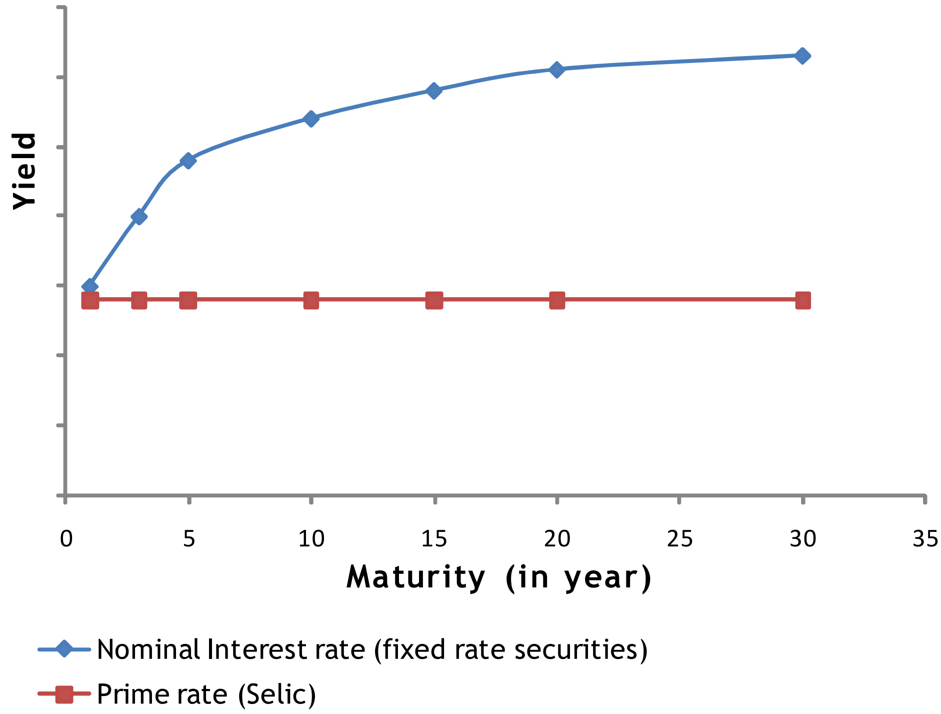
Particularly in relation to the yield curves, the scenarios are necessary in order to obtain the cost of each financing option. Although the cost of FPD financing depends primarily on the prime interest rate of the economy, each debt instrument has particularities, especially with regard to indexing factors, maturity and degree of liquidity.

The most basic case applies to floating rate bonds (Selic rate). The model presumes that they are sold at par or, in other words, their price is equal to face value. The cost is then defined by the Selic rate composed on a daily basis throughout the period, independently of its maturity.

Given the other alternatives available for FPD financing, the model has specific yield curves for each type of instrument. Although cost is based on the short-term interest rate, each debt instrument has its own particularities, for example, with respect to maturity and degree of liquidity. In this sense, a yield curve model is specified to obtain the nominal interest rate that will define the cost of fixed rate bonds, according to the maturity of the instrument to be issued, as shown in Figure 3.

[8] Nominal exchange rate is obtained by aggregating the differential between domestic and foreign inflation into real exchange rate.

Figure 3. Fixed-rate Curve



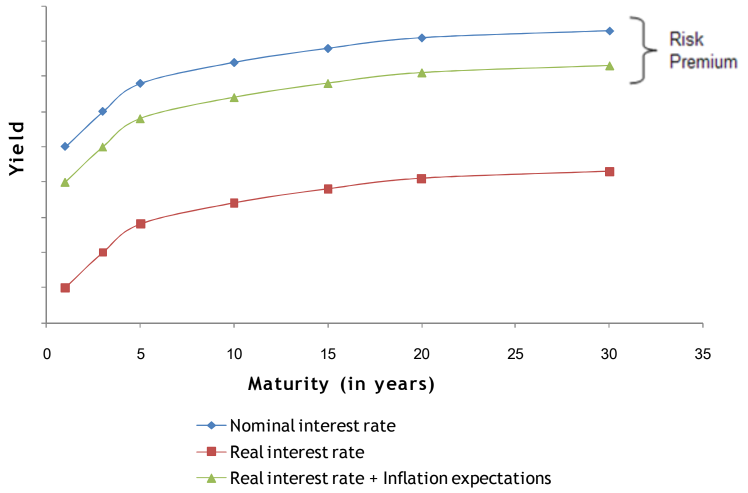
Source: National Treasury

In the Brazilian case, aside from fixed rate bonds, one should also specify yield curves for inflation-linked bonds and for those with linked to exchange rate variations (in the case of the external debt). The model is specified in such a way that the cost of these alternatives is related to the cost of fixed rate bonds with equivalent maturities, adjusted by a risk premium that will be explained below.

The risk premium structure of the model reflects how lower the returns on a security linked to inflation or exchange rate variations should be, compared to bonds with nominal fixed rate yield and equivalent maturities. The idea here is that the existence of a factor that protects real returns on the bond should be provided to the issuer through a lesser risk premium.

Figure 4 illustrates how yield curves are related in the model. The expected cost of inflation-linked securities is composed of the sum of the real interest rate - given by the specific yield curve used to price these securities - plus inflation expectations. This expected cost will be less than the average cost of fixed rate bonds in the presence of a positive inflationary risk premium, since the real returns of the holder of the indexed security are protected against unexpected interest rate variations.

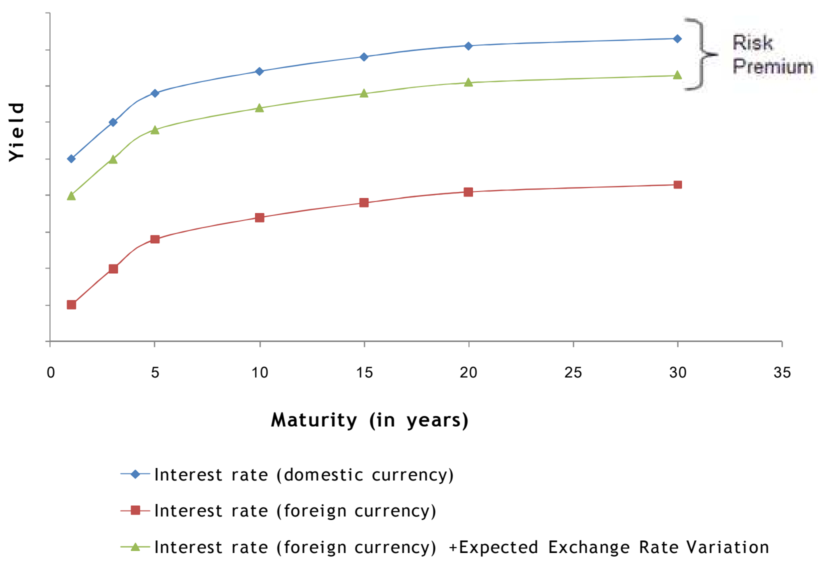
Figure 4. Inflation Risk Premium



Source: National Treasury

In a similar manner, as shown in Figure 5, there is also a direct relation between the yield curve in foreign currency and the curve for fixed rate securities. With respect to bonds denominated in foreign currency, the reason for the risk premium originates in the fact that the external investor desires a hedge against exchange rate fluctuations.

Figure 5. Exchange Risk Premium



Source: National Treasury

Following simulation of the scenarios for financial and macroeconomic variables, the cost of FPD financing is calculated, which depends on the issuance cost of each public bond plus variation of its indexing factor, when appropriate. Appendix 6.4 explains how this calculation is done.

2.4. Dynamics of the Debt

Once the scenarios have been defined, the following accounting identity is used as the starting point for deriving the dynamics of the NPSD and its relation to the FPD.

(1) $$D_t = X_t + M_t - F_t + L_t^{flutuante} + L_t^{IP} + L_t^{FX}$$

In this identity, the assets and liabilities included in NPSD (D) are grouped into four categories: FPD (X), monetary base (M), international reserves (F) and other net public sector liabilities[9] [indexed to floating rates ($L_t^{flutuante}$), inflation (L_t^{IP}), and exchange rate variation (L_t^{FX})].

Outstanding FPD in period t is equal to the outstanding volume in the previous period plus its carrying cost (c_t), less the primary fiscal result, less variation of the monetary base. Consequently, FPD evolves according to the following equation:

(2) $$X_t = X_{t-1}(1 + c_t) - S_t - \Delta M_t$$

With respect to the other components of NPSD, it is presumed that the monetary base remains constant as a proportion of GDP over time, while international reserves (equation 3)[10], as well as other net public sector liabilities (equations 4-6) start from an initial volume and evolve according to their returns.

(3) $$F_t = F_{t-1}(1 + r_t^{\text{reserves}})(1 + \Delta c\hat{a}mbio)$$

(4) $$L_t^{flutuante} = L_{t-1}^{floating}(1 + \text{Selic}_\text{t})$$

(5) $$L_t^{IP} = L_{t-1}^{IP}(1 + c_t^{IP})$$

(6) $$L_t^{FX} = L_{t-1}^{FX}(1 + c_t^{FX})$$

[9] FPD encompasses only federal government liabilities. To shift to the NPSD concept, one must consider that this debt includes nonfinancial public sector liabilities plus the Central Bank. Therefore, this concept includes the direct federal, state and municipal administrations, indirect administrations, the public social security system, nonfinancial government companies, as well as the Central Bank of Brazil. Additionally, as a net concept, NPSD deducts public sector financial assets (for example, international reserves, funds such as the Worker Support Fund and credits with financial institutions) from liabilities. Finally, intragovernmental debts (crossed relations) are excluded, in such a way that only the public sector debt with private agents is measured.

[10] The rate of return on international reserves ($r_t^{reserves}$)in the model may be different from the average cost of the external debt.

In which c^{IP} and c^{FX} are the carrying cost of the bonds indexed to inflation and denominated in foreign currency; and $r_t^{reserves}$) represents the rate of return of international reserves.

After substituting (2) – (6) in (1) and dividing the new equation by GDP, algebraic manipulations lead to the following formula to describe the trajectory of the NPSD/GDP ratio over time:

$$(7) \qquad d_t = x_{t-1}\frac{(1+c_t)}{(1+\gamma_t)} - s_t + \frac{m}{(1+\gamma_t)} - f_{t-1}\frac{(1+c_t^{\text{reserves}})}{(1+\gamma_t)} + l_{t-1}\frac{(1+c_t^l)}{(1+\gamma_t)}$$

In which:

$$c_t^{\text{reserves}} = (1 + r_t^{\text{reserves}})(1 + \Delta câmbio) - 1$$

$$l_t = l_t^{floating} + l_t^{IP} + l_t^{FX}$$

$$c_t^l = \left(Selic_t l_t^{floating} + c_t^{IP} l_t^{IP} + c_t^{FX} l_t^{FX}\right)/l_t$$

3. Optimal Composition Model Simulations

By means of an exercise, this section will demonstrate the application of the model described above to FPD. The following data and results are merely illustrative. In actual practice, aside from the simulations based on a fundamental set of parameters, the robustness of the conclusions is tested when analyzing the sensitivity of the model to variations in benchmark parameters.

At the same time, though the definition an optimal composition (benchmark) is based on inputs derived from simulations, it will depend on a wide-ranging discussion that gives due consideration to the feasibility of adopting a specific debt profile in a given temporal horizon, as well as an understanding of the interactions between debt management and such other economic policies as fiscal and monetary policy.

The first step in the simulation is to obtain the parameters and initial values[11] for the stochastic models and, with this, generate the macroeconomic scenarios. These parameters depend on the stochastic model adopted but, in general, can be described in terms of the average and volatility of the scenarios for each variable. The following tables show the scenarios generated by the models for the macroeconomic variables[12]:

[11] Since the model presupposes work in the stationary state, all of the macroeconomic variables remain in the range of their long-term averages. For this reason, the initial values are the same as the long-term values.
[12] The stochastic processes currently employed by the National Treasury are detailed in Appendix 6.1.

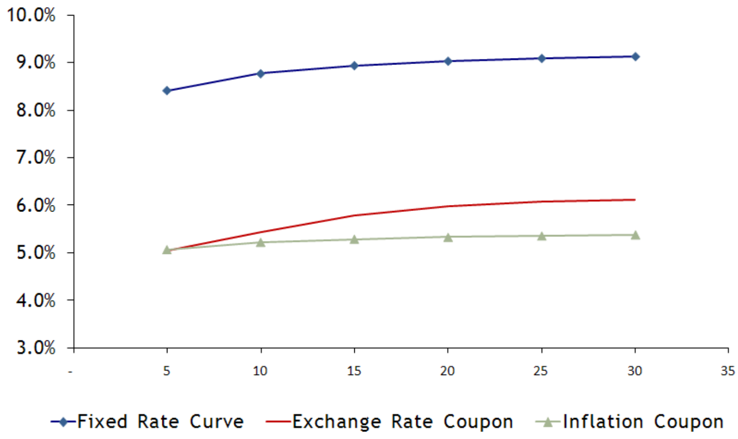
Table 2. Scenarios Generated (cumulative in 12 months)

Per Year	SELIC	IGP-M	CPI	Nominal Exchange Rate	Libor	GDP
Average	7.45%	2.98%	2.99%	2.84%	4.69%	4.50%
Standard Deviation	1.80%	1.11%	0.70%	14.28%	1.52%	2.33%
Percentile 5°	4.79%	1.16%	1.84%	-18.94%	2.51%	0.72%
Percentile 95°	10.67%	4.83%	4.15%	22.77%	7.48%	8.39%

Source: National Treasury

Calculation of the cost of FPD financing also depends on simulation of yield curves. In average terms, the figure below shows the cost of fixed rate bonds, the real yield curve (for inflation-linked bonds) and the yield curve in dollars (for bonds denominated in foreign currency). As already described, once the prices and, consequently, the cost of the fixed rate bonds have been defined, the cost of the inflation-linked bonds is that of the fixed rate bonds less inflation expectations, less an inflation risk premium. Analogously, the cost of exchange rate-indexed bonds is that of fixed rate bonds, less expectations of currency devaluation, less an exchange risk premium.

Figure 6. Fixed Rate Curve, Inflation Coupon and Exchange Rate Coupon



Source: National Treasury]

Table 3 shows the carrying cost of each financing operation for the selected instruments. In this case, aside from the rate of return indicated by the yield curve, the cost is calculated by adding in the variation of the indexing factor of the bond throughout the period. Considering a 10-year horizon, dispersion of the cost is also explained by the dynamics of debt refinancing during the simulations.

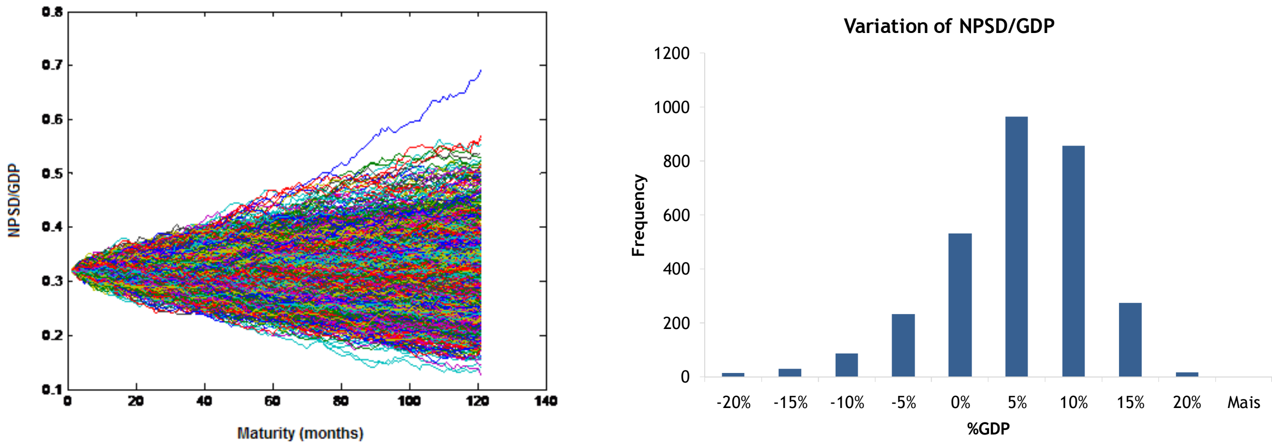
Table3. Financing Cost in 10 Years (cumulative in 12 months)

Per Year	Average	Standard Deviation	Percentile 5	Percentile 95
Fixed 1 year	7.77%	1.40%	5.55%	10.43%
Fixed 3 years	8.14%	0.84%	6.78%	9.75%
Fixed 5 years	8.40%	0.52%	7.55%	9.39%
Fixed 10 years	8.76%	0.16%	8.50%	9.06%
Inflation 10 years	8.21%	2.41%	4.07%	12.50%
Inflation 30 years	8.37%	2.40%	4.24%	12.66%
Exchange rate 10 years	8.24%	68.80%	-56.29%	151.80%
Exchange rate30 years	9.24%	68.91%	-55.91%	154.26%
Selic 5 years	7.51%	1.83%	4.63%	10.99%

Source: National Treasury

Once the costs of each instrument are obtained, the dynamics of the FPD and, after that, of the NPSD, are calculated for portfolios composed 100% of a specific instrument and for portfolios with pairs of securities in a proportion of 50%-50%. The figure below shows the trajectories of 3000 simulations of NPSD/GDP for a portfolio composed 100% of bonds linked to the Selic rate, together with a histogram with the distribution of the variation of the NPSD/GDP in these simulations.

Figure 7. Dynamics of the NPSD/GDP

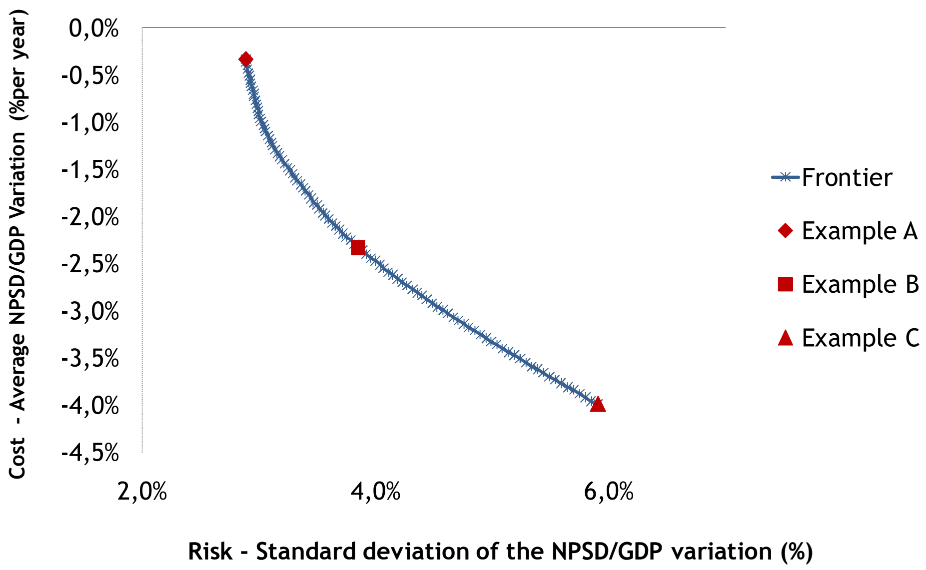


Source: National Treasury

In a manner similar to that presented above, for each simulated portfolio there are the distribution of the NPSD/GDP in the analysis period, its average, the standard deviation and the correlation matrix of the portfolios. Cost is defined as the average of the NPSD/GDP variation and the risk as its standard deviation. With this information, the efficient frontier is generated.

Utilizing the efficient frontier, the results are presented in a graph in the cost/risk space. The frontier may be generated with the nominal cost and risk or with a specific portfolio as reference (for example, the lowest risk composition of the simulations) and performing the calculations related to it. This frontier does not consider any analysis of factibility of the portfolios due to the particularities of the public bond market or other premises for debt management, since it is a frontier with a purely financial bias.

Figure 8. Efficient Frontier



Source: National Treasury

An FPD composition is associated to each point on the efficient frontier above. By way of example, the following Table shows the FPD profile that characterizes points A, B and C highlighted in Figure 8.

Table 4. Composition of the Portfolios

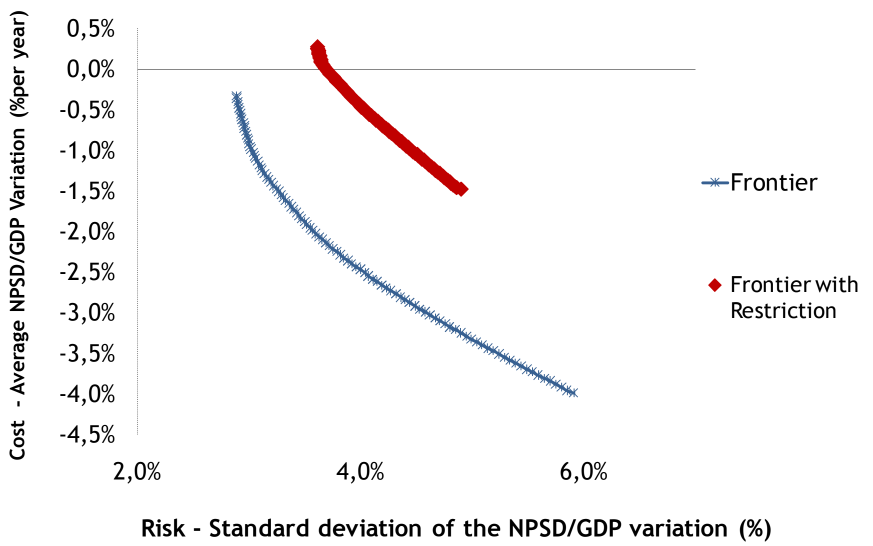
Examples	Portfolio Composition	
	Inflation	Exchange rate
A	100.0%	0.0%
B	50.0%	50.0%
C	0.0%	100.0%

Source: National Treasury

Another interesting characteristic of the model is the possibility of defining technical restrictions on compositions in the efficient frontier. This allows for greater flexibility, since there are some aspects of economic policy or the market that are not directly perceived initially, but that may be defined as restrictions on the efficient portfolio. One should recall that the efficient frontier is a function not only of the composition, but also of the maturity of the instruments. With this, the restrictions may involve any one of these two aspects.

One possible restriction is the existence of a limited demand for a specific type of instrument or, in other words, a limit of feasibility for some FPD compositions. The manager may also define a minimum value for the average maturity of the outstanding volume or a maximum value for the percentage of the debt maturing in 12 months, based on the management guidelines of that debt. The following figure presents a frontier with restrictions, as cited above:

Figure 9. Efficient Frontier with Restrictions

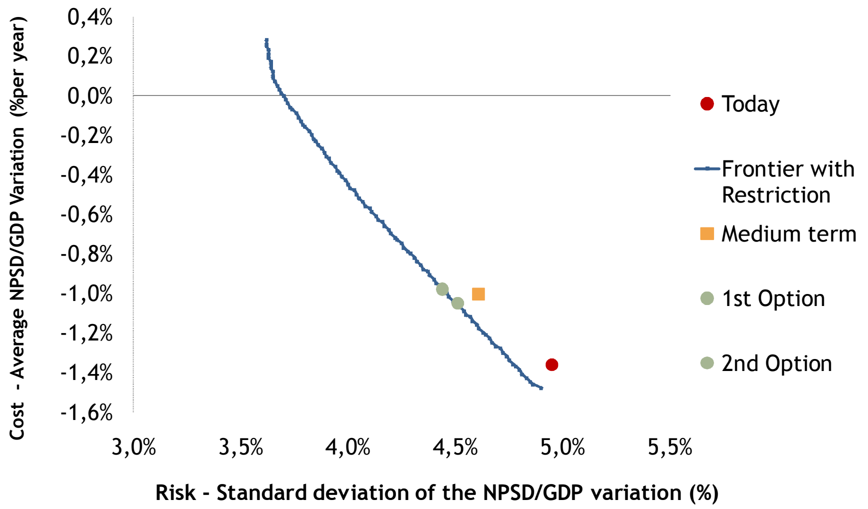


Source: National Treasury

The choice of a composition as benchmark for the public debt implies a choice of the risk that the government (and society) is willing to run and the cost that it is willing to pay in order to protect itself. Given a risk level, the composition of minimum cost may be obtained from the frontier. The debate based on results is wide-ranging and the choice of an optimal composition is viewed in the framework of strategic planning of public debt management.

In order to illustrate the latter point, the following figure shows the efficient frontier with restrictions and, given the parameters utilized, how a debt composition positions itself in relation to the frontier. In this case, one notes that there would be space to increase efficiency in debt management through alterations in the FPD composition toward some portfolio from the frontier.

Figure 10. Efficient Frontier and Medium-Term*



* Note: The references presented in this graph are merely illustrative and, therefore, do not necessarily correspond to the National Treasury's choice of options.
Source: National Treasury

Analysis of the path to be followed toward a specific optimal portfolio must be carried out within the framework of medium-term strategies, seeking to design a transition plan, with due consideration of current debt composition and its maturity structure and how fast convergence to the desired debt profile in the future could be. As an example, Figure 10 shows that the portfolio resulting from the transition strategy (medium-term) would represent a higher cost and lower risk than the current portfolio, while also positioning itself relatively closer to the efficient frontier.

4. Final Considerations

In the case of the Brazilian Federal Public Debt, the initial optimal composition proposal was published in the 2007 Annual Borrowing Plan - ABP[13]. These simulations of the model suggested that efficient management of FPD would be that which leads to an increased proportion of fixed rate securities and inflation-linked securities, in detriment to floating rate and exchange rate-indexed debt. More recently, refinement of studies on definition of the optimal composition (benchmark) based on FPD management objectives and evaluation of risks, restrictions and opportunities in coming years, led to definition of the desired composition, which is presented in the 2011 ABP in the form of indicative long-term limits, as shown in the following table:

Table 5. Optimal Long-Term FPD Composition

	Lower limit	Upper limit
Fixed rate	40%	50%
Inflation-linked	30%	35%
Floating rate	10%	20%
Exchange rate	5%	10%

Source: National Treasury

As stressed in the 2011 ABP, the prescription for seeking the composition described above should be qualified. In the first place, it must be viewed as a guideline to be achieved gradually, without generating pressures that result in excessive transition costs. Secondly, FPD composition must not be sought in a manner uncoordinated with its maturity structure. Lengthening of the average FPD maturity and, more specifically, of that of fixed rate bonds is a necessary condition for the composition suggested for FPD to result in efficiency gains and risk reductions (this result is also derived from the simulations).

In the third place, the cost of the composition change must be permanently monitored. Significant oscillations in the relative prices of the different instruments included in these strategies, due above all to changes in the risk premiums, may result in adjustments in the benchmark portfolio for FPD.

Finally, though these limits provide a current guide for defining strategies, they must also reflect possible restrictions related to the development stage of financial markets in Brazil, to the investor base profile and to the demand and future liquidity outlook for public bonds. The speed of convergence from the current FPD composition to that indicated in Table 5 will depend on overcoming some of these restrictions.

[13] The annual borrowing plans can be found at http://www.tesouro.fazenda.gov.br/english/public_debt/annual_borrowing_plan.asp

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